GNSS carrier phase processing using Modified Ambiguity Function Approach

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SUMMARY

This paper presents an algorithm for GNSS carrier phase positioning based on some properties of Ambiguity Function Method. There is no stage of ambiguity search/resolution in the presented approach, since there is no ambiguity parameter in the proposed adjustment model. The integer nature of the ambiguities is ensured in the results through the least squares adjustment with condition equations in the functional model. Therefore the proposed method is robust to cycle slip effect. Also, an appropriate function for the condition equations is presented and tested here. The new approach requires subsequent processing of the linear combinations of GNSS signals in the cascade adjustment. The presented numerical tests were performed using in-house developed GINPOS software and GPS data sample collected by Polish GNSS network - ASG-EUPOS.

1. INTRODUCTION

There are two groups of unknowns in the Global Navigation Satellite System (GNSS) carrier phase data processing. The first group contains real-valued unknowns (e.g. coordinates); the second one consists of carrier phase initial integer ambiguities. Currently, the most popular algorithms of precise positioning are based on the three stage procedure of the Integer Least Square Adjustment (Teunissen 1995):

- float solution (neglecting integer nature of some parameters)
- ambiguity resolution (AR) (search for integer ambiguity parameters)
- fixed solution (introducing integer ambiguities).

In the first stage, the solution in real number domain for all unknowns is obtained. In the second stage, the procedure of searching for the integer ambiguities with the validation process is carried out. Finally, in the third stage, the remaining real-valued unknowns (e.g., coordinates) are solved again by introducing fixed integer ambiguities. Thus this (classic) approach requires the additional stage of the ambiguity resolution. The values of the ambiguities are computed explicitly which in turn, requires cycle slip detection and repairing. The procedure of searching for the most probable values of the ambiguities is performed in previously determined (limited) search space. Currently, as the most efficient method of AR is considered the LAMBDA method (Teunissen 1995; Teunissen 1995; Chang et al. 2005).

Ambiguity Function Method (AFM) is one of the alternative approaches. This method takes advantage of certain properties of the chosen periodic functions, which have known values for the integer arguments (Counselman and Gourevitch 1981; Remondi 1990; Mader 1990; Han and Rizos 1996). Although this method has good mathematical bases, it is less popular than classic approach. This paper presents a new approach for GPS carrier phase positioning based on certain properties of AFM. The presented approach is called MAFA (Modified Ambiguity Function Approach) (Cellmer et al. 2010). The MAFA algorithm ensures the condition of parameter "integerness" without the necessity for the additional stage of the integer search. It is based on the least squares adjustment (LSA) algorithm with condition equations in the functional model. In order to derive such functional model an appropriate formula for function of the condition equation is used and given below.

However, the derived functional model is relatively weak. Hence, in order to assure the appropriate convergence of the computational process, linear combinations (LC) of L_1 and L_2 GPS carrier phase observables are applied in the cascade adjustment, e.g., the computations are performed successively for different linear combinations (Han and Rizos 1996; Jung and Enge 2000).

2. THE CONDITIONAL EQUATION OF THE CARRIER PHASE OBSERVATION

The simple form of the observation equation for double differenced (DD) carrier phase observable can be written as follows (Leick 2004; Hoffman-Wellenhof et al. 2008; Teunissen and Kleusberg 1998):

$$\Phi + v = \tfrac{1}{\lambda}\rho(X_c) + N \quad (1)$$

where:

Φ – DD carrier phase observable (in cycles)
λ – signal wave length
v – residual (measurement noise)
X_c – receiver coordinate vector
$\rho(X_c)$ – DD geometrical range
N – integer number of cycles (DD initial ambiguity)

Each term in equation (1) is expressed in the units of carrier cycles. There are two groups of parameters in this equation. The first group consists of three real-value receiver coordinates (included in ρ) and the second group consists of an integer-value DD ambiguity (N).

In general case the first group may contain additional parameters: ionospheric and tropospheric delays, etc. However, it does not affect the course of the computational process of the presented methodology.

The equation (1) can be rewritten as:

$$\Phi + v - \tfrac{1}{\lambda}\rho(X_c) = N \quad (2)$$

For the simplicity, the term $\rho(X_c)$ is represented as ρ in the subsequent equations.

Typically, carrier phase measurement accuracy is of about 0.01 cycle (Hofmann-Wellenhof et al. 2008). Thus the residual values should be much lower than half a cycle.

Hence, taking into account the integer nature of the ambiguity parameter N, the equation (2) can be rewritten in the following form:

$$\Phi + v - \tfrac{1}{\lambda}\rho = \mathrm{round}\left(\Phi - \tfrac{1}{\lambda}\rho\right). \tag{3}$$

or

$$v = \mathrm{round}\left(\Phi - \tfrac{1}{\lambda}\rho\right) - \left(\Phi - \tfrac{1}{\lambda}\rho\right). \tag{4}$$

where *round* is a function of rounding to the nearest integer value. Note that the residuals (4) taken into account the integer nature of ambiguities N. LSA procedure requires linearization of the right side of the equation (4). For the purpose of linearization, in the papers Cellmer (2009) and Cellmer et al. (2010), the authors proposed differentiable function in the place of the term on the right side of the equation (4). That function, however, had some points of discontinuity. Here, a new differentiable and continuous function is proposed:

$$\Psi = \mathrm{round}(s) - s = \begin{cases} -\dfrac{1}{\pi}\arcsin[\sin(\pi s)]\ \mathrm{for}\ s \in \{s : \cos(\pi s) \ge 0\} \\ \dfrac{1}{\pi}\arcsin[\sin(\pi s)]\ \mathrm{for}\ s \in \{s : \cos(\pi s) < 0\} \end{cases}, \tag{5}$$

where s is an auxiliary variable:

$$s = \Phi - \tfrac{1}{\lambda}\rho. \tag{6}$$

The derivative of Ψ is:

$$\frac{\partial \Psi}{\partial X_c} = \frac{\partial \Psi}{\partial s}\frac{\partial s}{\partial X_c}. \tag{7}$$

It is easy to proof that:

$$\frac{\partial \Psi}{\partial s} = -1,\ \text{for}\ \ s \in R \tag{8}$$

and

$$\frac{\partial s}{\partial X_c} = -\frac{1}{\lambda}\frac{\partial \rho}{\partial X_c}. \tag{9}$$

Hence, after Taylor series expansion:

$$v = \Psi = \frac{1}{\lambda}\frac{\partial \rho}{\partial X_c}\Big|_{X_{c0}} X + \Psi(X_{c0}) \tag{10}$$

or

$$v = \frac{1}{\lambda}\left(\frac{\partial \rho}{\partial x}\Big|_{X_{c0}} dx + \frac{\partial \rho}{\partial y}\Big|_{X_{c0}} dy + \frac{\partial \rho}{\partial z}\Big|_{X_{c0}} dz\right) + \mathrm{round}\left(\Phi - \frac{1}{\lambda}\rho_0\right) - \left(\Phi - \frac{1}{\lambda}\rho_0\right), \tag{11}$$

where, $X_{c0} = [x_0, y_0, z_0]$ is the vector of approximate coordinates and ρ_0 is the corresponding approximate double differenced geometric distance between the station and the satellite whereas: dx, dy, dz are elements of the unknown parameter vector X.

The residual equations (11) are formed for each of n DD carrier phase observations. General formula of the residual equations can be shown in the following form:

$$V = \tfrac{1}{\lambda} AX + \Delta, \tag{12}$$

with:

$$X = [dx,dy,dz]^T \tag{13}$$

$$A = \begin{bmatrix} \frac{\partial\rho_1}{\partial x} & \frac{\partial\rho_1}{\partial y} & \frac{\partial\rho_1}{\partial z} \\ \frac{\partial\rho_2}{\partial x} & \frac{\partial\rho_2}{\partial y} & \frac{\partial\rho_2}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial\rho_n}{\partial x} & \frac{\partial\rho_n}{\partial y} & \frac{\partial\rho_n}{\partial z} \end{bmatrix} \tag{14}$$

$$\Delta = \mathrm{round}\left(\Phi - \tfrac{1}{\lambda}\rho_0\right) - \left(\Phi - \tfrac{1}{\lambda}\rho_0\right) \tag{15}$$

where:

V – residual vector ($n\times1$),
X – parameter vector (increments to a priori coordinates vector X_0),
A – design matrix ($n\times3$),
Δ – misclosures vector ($n \times1$),
n – number of DD observations.

3. ADJUSTMENT PROBLEM

Adjustment problem can be formulated as:

$$V^TPV = \min \tag{16}$$

where:

V – residual vector, equation (12),
P – weight matrix.

The solution of this problem is the following parameter vector:

$$X = -\lambda(A^TPA)^{-1}A^TP\Delta \tag{17}$$

together with its covariance matrix:

$$C_X = \sigma_0^2\lambda^2(A^TPA)^{-1} \tag{18}$$

The ambiguity parameters are not present in the derived adjustment model. Nevertheless, the above formulation gives results that fulfill the condition of the integer ambiguities. Therefore, there is no need to deal with, e.g., cycle slip effects.

4. ALGORITHM OF THE CASCADE ADJUSTMENT WITH SUBSEQUENT LINEAR COMBINATIONS

In case of equations (12) written for phase observations on the first frequency (GPS L_1), the objective function of the LSA procedure (16) has many local minimums. In case of insufficient quality of the parameter vector approximation, the solution may be found in any local minimum instead of te global one (where the actual correct solution exists). The final correct solution depends on sufficiently close approximation of parameter values X.
In order to solve this problem, different LC of L_1 and L_2 observations with integer ambiguities and longer wavelengths may be applied. Table 1 presents the linear

combinations used in the proposed method, along with their wavelengths (Han and Rizos 1996).

Table 1 Linear combinations of L1 and L2 signals with integer ambiguity

LC#	i	j	λ [m]
1	-3	4	1.6281
2	1	-1	0.8619
3	1	0	0.1903

The above linear combinations were chosen based on the analyses of theoretical properties of these combinations (Han and Rizos 1996) and on the basis of large number of numerical tests performed.

In the computation process the adjustment is carried out for observation sets created for the linear combinations. The adjustment takes place successively in order listed in Table 1, namely starting from LC with the longest wavelength.

5. NUMERICAL EXAMPLE

Test surveys were performed on May 8th, 2007, 8:30:00-10:30:00 UT on 25 km baseline, with 30-second sampling rate. The selected stations are located in southern Poland and belong to the national active geodetic network – ASG-EUPOS. Figure 1 illustrates the location of the test baseline. Station TARG served as a simulated user receiver. Two hour data set was divided into 24, five minutes (10 epochs) long sessions. The sessions were processed using the proposed methodology. The first correct solution was usually obtained after processing just 1 epoch of data. Next solutions were obtained adding the data from the consecutive epochs in the sequential adjustment. Reference coordinates were derived with Bernese software using 10-hour data set (Dach at al., 2007).

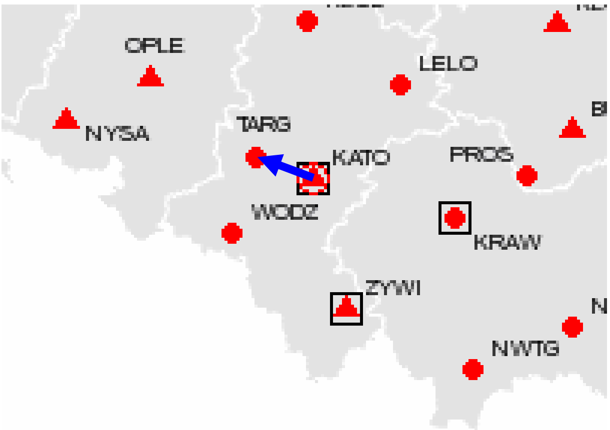

Fig. 1 ASG-EUPOS stations in south part of Poland and the baseline (25 km) analyzed in the experiment

Figure 2 presents N, E and U component residuals, with respect to "true" position from Bernese.

In the presented example only two sessions (8:50-8:55 and 8:55-9:00) did not give a good solution. In most of the sessions the solutions converged already in the first epoch. Only in two sessions the solutions stabilized later: the session starting at 8:40 converged in the second epoch and the session starting at 9:15 - in the fourth epoch. The example was carried out only in order to test the feasibility of the MAFA method. The data sample size was insufficient to give an opportunity to draw any conclusions concerning its qualitative properties. Therefore acceptable is the statement that the results obtained so far are promising and the method is worth of the further research.

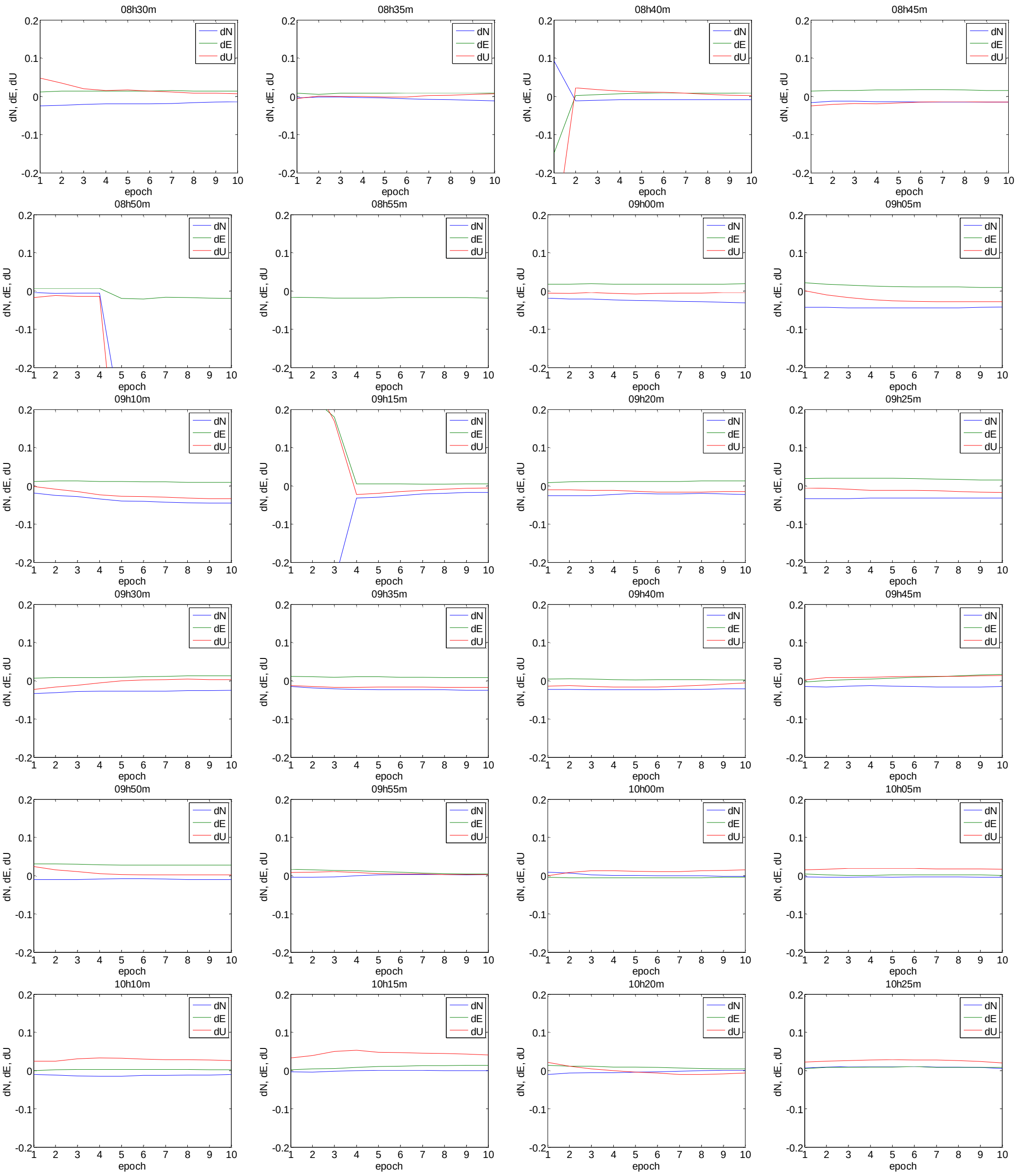

Fig. 2 N, E and U component residuals in [m] with respect to the reference position.

6. CONCLUSIONS

The new method for carrier phase-based GPS positioning was presented in this paper. It has been shown that the new method enables precise GPS positioning without the necessity of explicit computation of the carrier phase DD ambiguities, although the condition of their "integerness" is fulfilled. The advantages of the presented method are: simplicity and robustness to cycle slip effects. The tests show high efficiency when processing short observational sessions over 25-km baseline using linear combination of L1 and L2 signals. Hence, in our opinion, the proposed methodology may be successfully used for carrier phase GPS data processing in geodetic applications. However, further research and tests are required in order to fully validate the proposed approach.

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