

# Mesoscale anisotropy of GPS slant delay

Mariusz. Figurski, Marcin Gałuszkiewicz, Paweł Kamiński, Krzysztof Kroszczyński  
Military University of Technology  
[mfigurski@wat.edu.pl](mailto:mfigurski@wat.edu.pl), [kkroszczyński@wat.edu.pl](mailto:kkroszczyński@wat.edu.pl)

## Abstract

The paper presents the results of research concerning GPS slant delay determination using data from COAMPS (Coupled Ocean/Atmosphere Mesoscale Prediction System) non-hydrostatic model of the atmosphere which is run on IA64 Feniks computer cluster in the Department of Civil Engineering and Geodesy of the Military University of Technology. The aim of the numerical experiment was to determine temporal and spatial relationships of the GPS slant delay for small angles of elevation in adverse weather conditions. The research was conducted for the period from April 14, 2008 to April 20, 2008 when vast zones of precipitation related with active atmospheric fronts were observed over Europe. Forecasted fields of atmospheric state parameters generated by the model were applied to determine 3D refractivity fields. In consequence, the slant delay is the result of integrating the ray (eikonal) equation for the spatial function of atmospheric refractivity along the GPS wave propagation path. The results show that for small values of the elevation angles the slant delay is anisotropic and depends on the observation direction. It varies both with space and time. Due to interaction between microwaves and magnetic dipole momentum of water, slant delay includes spatial and temporal information concerning heterogeneity of atmospheric humidity distribution related especially with cloud systems.

**Keywords:** slant delay, non-hydrostatic mesoscale model, ray tracing, anisotropy.

## Introduction

Precise location by means of GPS technology has to take into consideration measurement corrections related with atmospheric impact on electromagnetic waves propagation from the satellite to the receiver. Atmosphere slows down the signal and consequently makes the real path of the ray from the satellite to the receiver longer than the geometric path. This phenomenon is called refraction or tropospheric delay. Nowadays various models of refraction are used due to nondispersivity of the troposphere for GPS frequency waves. Zenith Total Delay (ZTD) models dominated until recently. Two parts are distinguished here: the dry one (hydrostatic) determined from the pressure and temperature at the site and the wet one – more complicated to determine - depending on the spatial distribution of water vapor. ZTD is used for determination of Slant Total Delay (STD), i.e. delay for any angle of GPS satellite observation. Mapping functions used for this purpose depend significantly on meteorological models. The coefficients for the Niell's functions were obtained empirically using US Standard Atmosphere 1966 data while for the Isobaric Mapping Functions (IMF) – using NCEP (National Center for Environmental Prediction) numerical meteorological model [1]. Recent research (2006) concentrated on tropospheric mapping functions (Boehm, J., B. Werl, and H. Schuh [2]) using data from the European Centre for Medium-Range Weather Forecasts model (ECMWF). Coefficients of these functions (Vienna Mapping Functions - VMF) result from applying the ray tracing method. Similarly, R. Eresmaa and H. Järvinen of the Finnish Meteorological Institute [3] use the mesoscale High Resolution Limited Area Model (HIRLAM) to determine the STD. The model uses boundary conditions from the ECMWF. The paper presents research conducted using data from COAMPS ver.3.1 non-hydrostatic

mesoscale model of the Naval Research Laboratory [4,5]. This model enables various parameterization of physical processes in the atmosphere (e.g. water phase transformations) which is more adequate than the used in hydrostatic models. Using models like COAMPS or WRF is far-reaching because of their continual development. Plans for COAMPS development include increasing the upper limit level of the model to 100 km (currently 30 km). Such an extension is important to GPS signal propagation modeling, especially for numerical methods of the eikonal solving. WRF model is actively developed by many powerful meteorological institutions e.g. National Oceanic and Atmospheric Administration, National Centers for Environmental Prediction, Air Force Weather Agency etc. High resolution non-hydrostatic mesoscale models enable to locate anisotropic distribution of water in its various phases. They enable to determine heterogeneity of atmospheric refraction fields (refraction coefficient, propagation speed) of GPS electromagnetic waves which can be used to investigate and simulate propagation of microwave radiation in the atmosphere.

### **Meteorological conditions**

The main aim of the numerical experiment was to determine temporal and spatial relationships of the GPS slant delay for small angles of elevation in adverse weather conditions. The research was conducted for the period from April 14, 2008 to April 20, 2008 when vast zones of precipitation related with active atmospheric fronts were observed over Europe (fig.3). These conditions (real meteorological situations) are presented by cloud systems observed in selected IR satellite images (8.3 – 9.1  $\mu\text{m}$ ) of the MSG2 (Dundee University Archive). Indicated areas (fig.3) are related to 13, 4.3 and 1.4 km telescopic nested grid used in calculations. The images are presented in the projection of the mesoscale model.

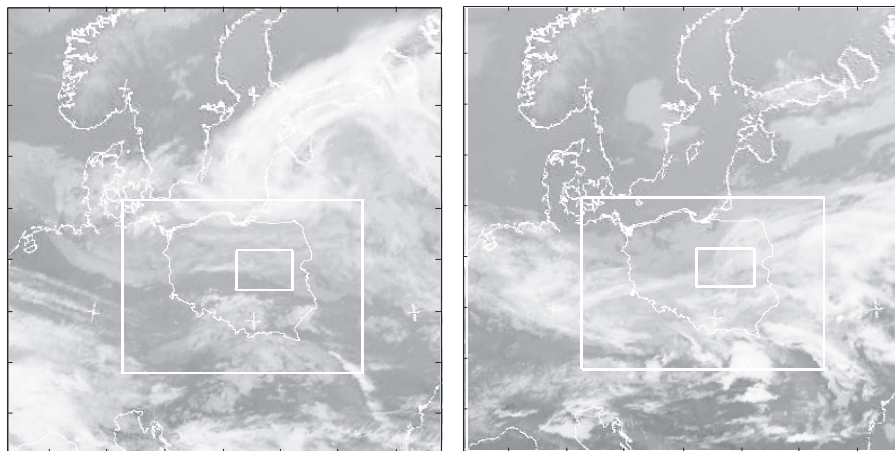


Fig.3. Cloud systems observed in IR satellite images of the MSG2 (17, 19.04.2008; 6:00 p.m.)

In our paper we used the model weather conditions. Note that the precipitation zones (fig.4) and cloud systems generated by the model are correlated with the ones observed in the satellite images. It is the result of data assimilation process, i.e. of the assessment of the initial state of the atmosphere by optimal application of information from various sources. Determination of the initial condition for the numerical prediction is the basic aim of data assimilation. Data assimilation is the subject of our separated investigations, which are related to necessity of maximum accordance between real meteorological conditions and conditions from forecasts of mesoscale model (we have also started research related with GPS data assimilation). For presented weather conditions w 3D refractivity fields were determined.

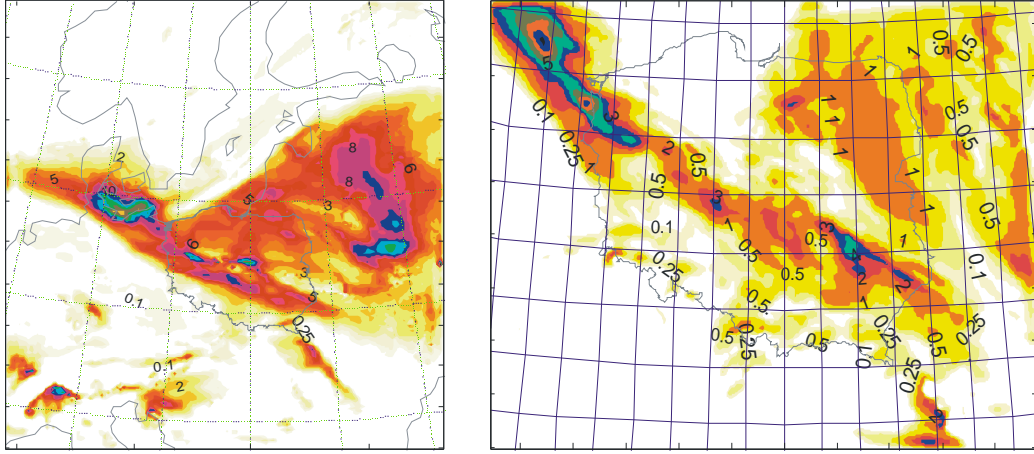


Fig. 4. Model weather conditions – stable precipitation ([mm/3h], 17.04.2008; 6:00 p.m.) for areas with grid - 13, 4.3 km

### Refraction

For refraction determination 3D fields of model COAMPS forecasted parameters i.e.: potential temperature –  $\theta$ , total (Exner) pressure -  $p$  and specific humidity –  $q$  are used. The grid of the “horizontal” computational surfaces of the model is rectangular and related to the Lambert Conformal projection. Nonlinear vertical coordinate is of  $\sigma$  type [5] and is defined as:

$$\sigma = H(z - z_s)/(H - z_s) \quad (1)$$

where  $H$  and  $z_s$  correspond to the depth of the atmosphere and the value of terrain height at the grid point (fig.1). The depth  $H$  is 31.5 km i.e.  $\sim 10$  hPa. The surfaces in the bottom part of the model of the atmosphere mimic orography. 3D field of refractivity  $N$  (refractive index –  $n$ ) is determined for 31 (40) computational levels, 39, 13, 4.3, 1.4 km grids and 1-hours intervals.

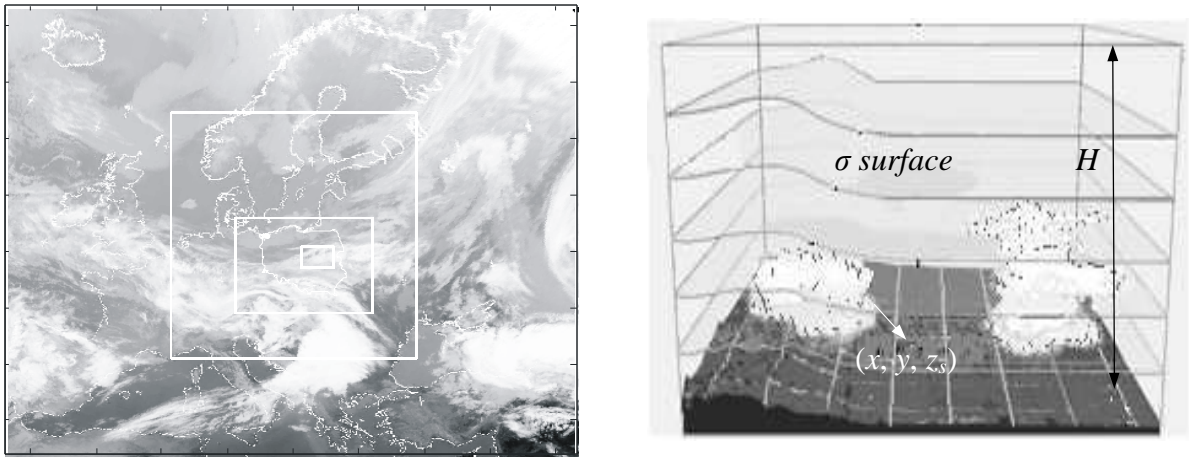


Fig.1. Computational areas and  $\sigma$ -surfaces of the COAMPS mesoscale model.

3D refractivity field  $N$  (fig. 2) is derived from the following equation:

$$N = k_1 \frac{p_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2}, p_d = p - e, e = qp/(0.622 + 0.378q) \quad (2)$$

where:  $p_d$ , and  $e$  – partial pressures of dry air and of water vapor,  $T$  – temperature and  $q$  specific humidity of the atmospheric air determined on computational surfaces of the model,  $k_1, k_2, k_3$  – constants determined experimentally [6].

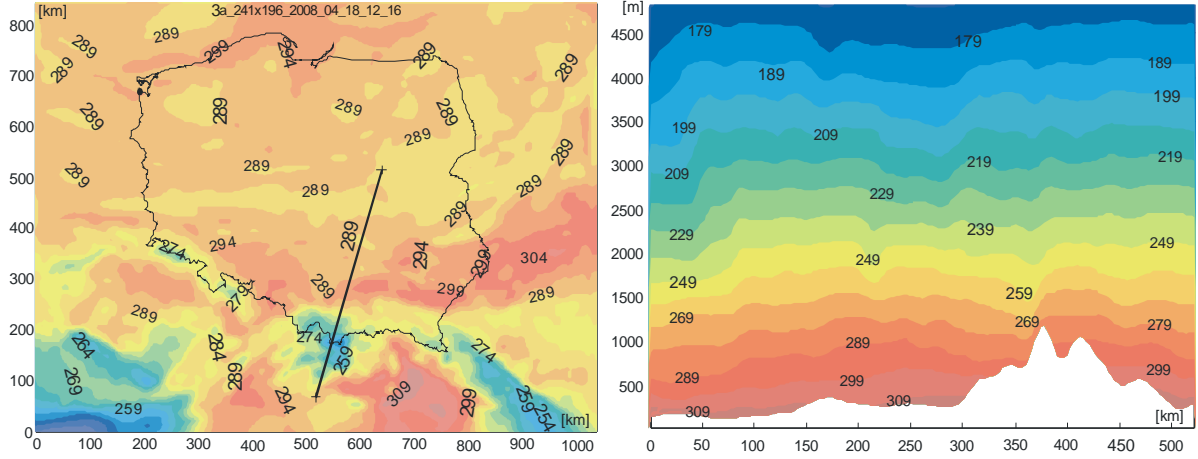


Fig.2. 2D refractivity field  $N$  on surface of 8th  $\sigma$  model level and vertical intersection of refractivity 3D field made along marked line.

### Slant delay

Slant delay – SD is a result of integrating the refractivity  $N$  along the GPS wave propagation path –  $ds$  from the receiver to the satellite:

$$STD = \int (n-1)ds = 10^{-6} \int Nds \quad (3)$$

The signal trajectory is a solution of the following geometric optical ray equation system [7]

$$d\mathbf{r}/ds = \mathbf{v}/n, \quad d\mathbf{v}/ds = \nabla n \quad (4)$$

where:  $\mathbf{r} = [x(t), y(t), z(t)]$  is the ray vector with respect to a selected reference point i.e. GPS station coordinates,  $\mathbf{v}$  – vector tangent to the trajectory at point  $\mathbf{r}$ .

The equation system (4) is solved by means of the Runge-Kutta type methods for a known field of the  $n$  refractive index and initial conditions related with the GPS station location and vector  $\mathbf{v}$  determined of the elevation angles of the satellites observed from the station. Due to discrete and nonlinear nature of the computational grid of the model, the index  $n$  and its gradient  $\nabla n(\mathbf{r})$  are results of two-step approximation in every computational step. In the first step, fields from model surfaces of equal altitudes  $z$  with respect to the terrain are approximated onto surfaces lying at the same altitude with respect to the sea level. The number of the surfaces is twice the number of the model levels  $\sigma$ . In the second step, values of  $\nabla n(\mathbf{r})$  are determined at the internal points of the approximated grid cells by means of polynomial trilinear interpolation. Polynomial structure of  $n$  enables easy refraction computation and 3D refraction gradient at a trajectory point that belongs to such a cell and to determine the next trajectory point according to system (4). This process is repeated until the ray leaves the model atmosphere. SD is determined by means of numerical integration using the ray points coordinates and refraction values at the points.

### Anisotropic character slant delay

The Matlab module was used to investigate the anisotropic character of the slant delay. The slant delay was determined for selected elevation angles  $\xi$  with  $1^\circ$  and  $5^\circ$  intervals in the ranges from  $3^\circ$  to  $10^\circ$  and  $10^\circ$  to  $90^\circ$  and azimuths  $\phi$  from  $0^\circ$  to  $350^\circ$  with  $10^\circ$  interval for selected points of the mesoscale model areas (spatial process of scanning the atmosphere). For examples the area - 1885 km by 2197 km (grid - 2) is sufficient to determine slant delay distributions for rays of  $3^\circ$  elevation e.g. for Poland (fig.5). The lengths of the rays projections are close to the value of:  $R = H/\text{tg}(3^\circ) \sim 570$  km -  $H$  is the depth of the atmosphere.

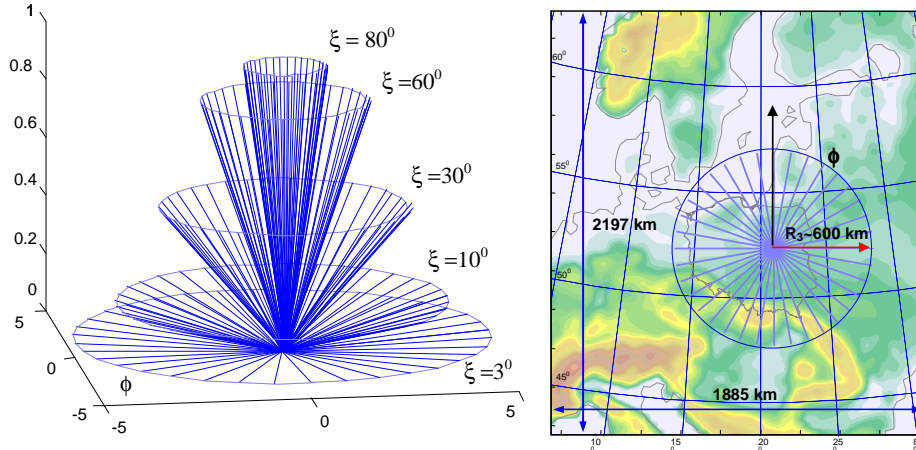


Fig.5. Scheme of computation of the slant delay corresponding to spatial process of scanning the atmosphere.

### Slant delay characteristics

According to the assumed scheme of computation, the distribution of the slant delay may be presented in the form of the following relation

$$\tau = \tau(\mathbf{x}, \xi, \phi, t) \quad (5)$$

They are multidimensional functions of station location  $\mathbf{x}$ , scanning angles  $\xi$ ,  $\phi$  and forecast time  $t$ . Since determination of the values of  $\tau$  for all nodes of the computational grid  $\mathbf{x}$  was not possible at the current state of the research, the point characteristics, i.e. determined for specific locations  $\mathbf{x}$  were considered. The presented data concern the ASG EUPOS MUT2 station (fig.6).

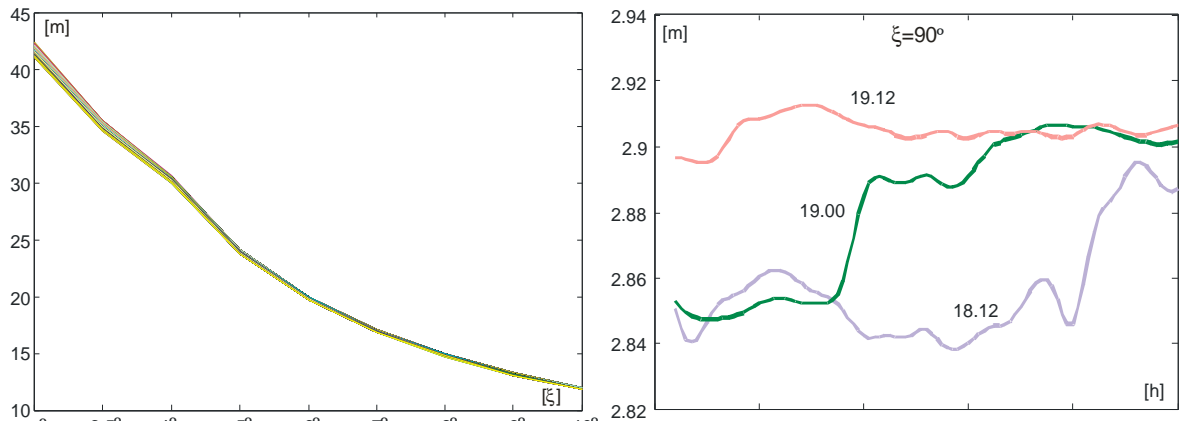


Fig.6. Dependence of slant delay on elevation angle  $\xi$  and temporary dependence of zenithal time delay for the MUT2 station (18, 19.04.2008). 00, 12 – forecast start time for 19.04.2008.

Relations determined according to the following formulas were used to investigate the anisotropy of the slant delay distribution:

$$\tau_{d1} = \tau - \tau_{\min}, \quad \tau_{d2} = \tau - \tau_{t=const} \quad (6)$$

where:  $\tau_{d1}$  or  $\tau_{d2}$  is the difference between  $\tau$  and its minimum value for chosen values of elevation angles  $\xi$  and time or the difference between  $\tau$  and its value for the chosen reference time ( $t = const$ ).

Introducing relative characteristics enables to better represent anisotropic (tensor) distribution of slant delay. The figures (6 – 8) show that for small values of elevation angle  $\xi = 3^\circ - 10^\circ$ ,  $\tau_d$

( $t$ ) depends on observation direction. In this case the GPS wave takes longer to pass the model atmosphere. The probability that it interacts on its way with water causing the delay is greater. The investigation results show that the azimuthal difference which is the measure of the delay anisotropy for taken into consideration adverse weather conditions may be for  $\xi > 3^\circ$  of the order one meter.

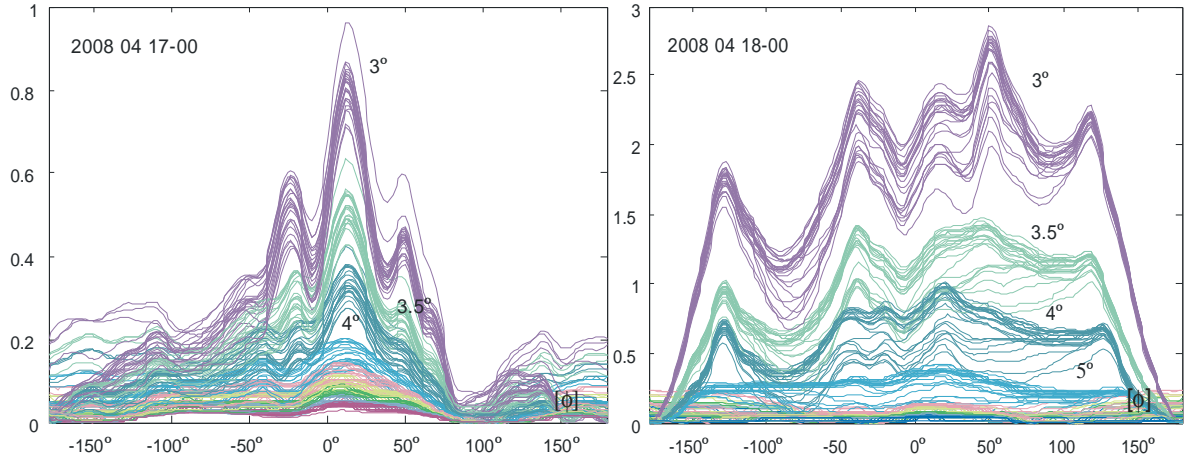


Fig.6. Azimuthal and temporal variability of slant delay  $\tau_{d1}(\mathbf{x} = \text{const}, \xi, \phi \in (-180^\circ, 180^\circ), t)$  for fixed elevation angles  $\xi = 3^\circ - 7^\circ$ , and location  $\mathbf{x}$  corresponding to MUT2 station.

24 hour variability of slant delay may be observed on (fig. 6). Sets of bundles (for fixed elevation angles  $\xi = 3^\circ - 7^\circ$ ) were created from 24 azimuthal courses for each hour of mesoscale model forecast. Figure 6 display also influence of atmospherical conditions change on SD after one day of forecast. It can be noticed that this change caused the almost twofold growth of differences  $\tau_{d1}$ . Anisotropy of distribution SD can be shown in other way by using 3D or 2D graphs (fig.7,8). Such visualization make spatial distribution of  $\tau_{d1}$  or  $\tau_{d2}$  analysis for GPS stations situated on area of model easier (fig.5).

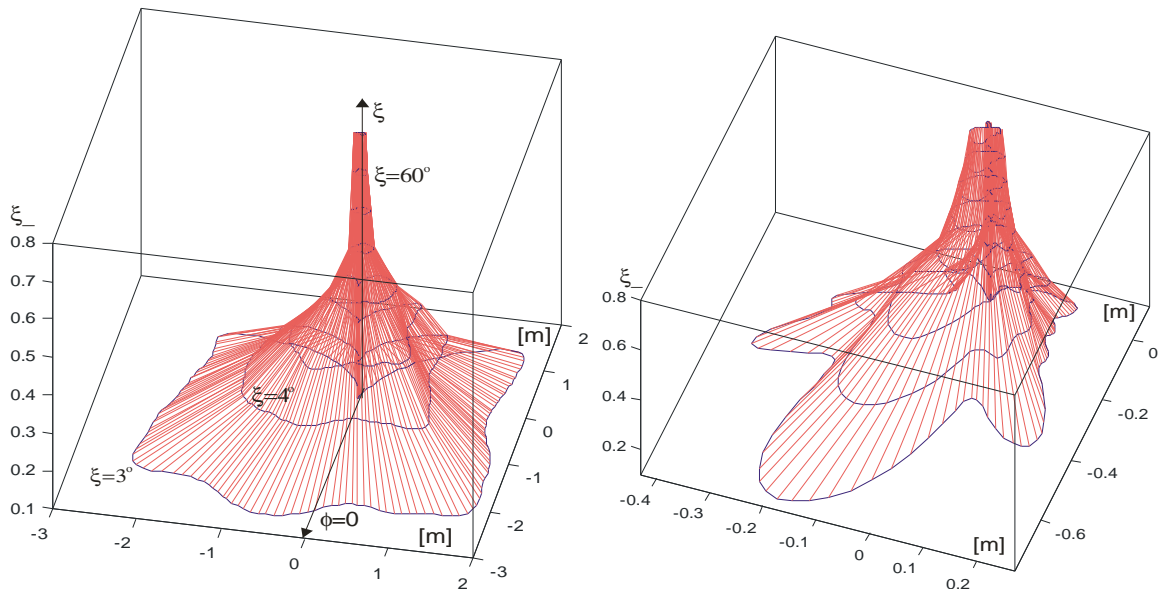


Fig.7. 3D anisotropic distributions  $\tau_{d1} = \tau - \tau_{\min}$  of slant delay  $\tau$  (difference between  $\tau$  and its minimal value),  $\phi = 0$  – axis of zero azimuth,  $\xi_{-}$  – conventional value of angle of elevation ( $\xi_{-} = 0 - \xi = 0^\circ$ ,  $\xi_{-} = 1 - \xi = 90^\circ$ ).

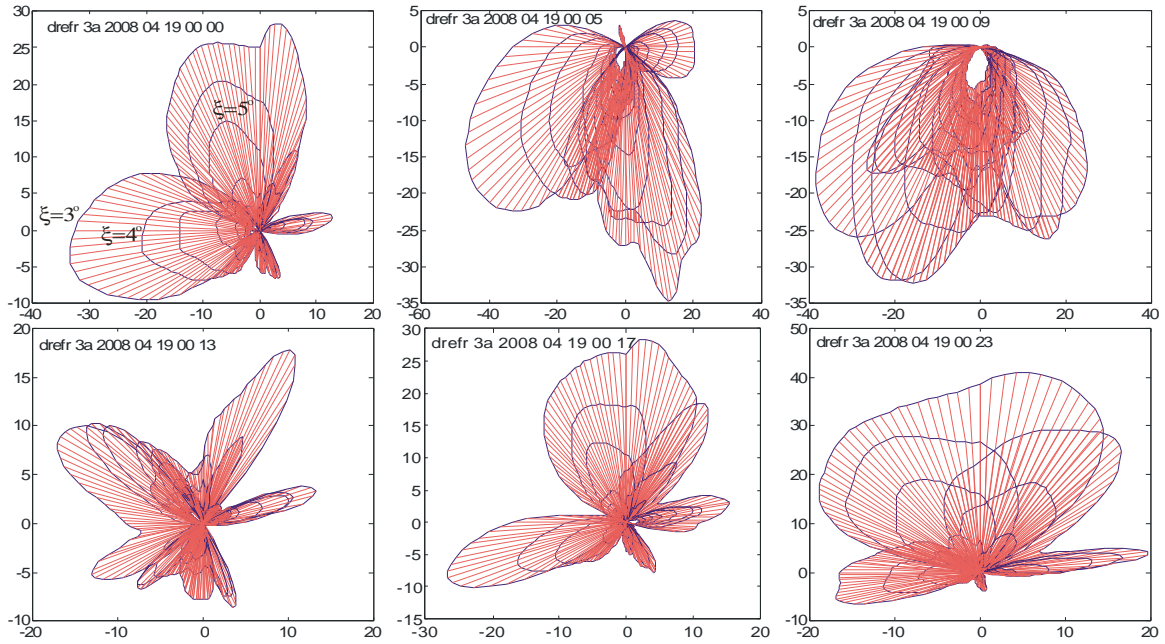


Fig.8. 2D anisotropic distributions  $\tau_{d2} = \tau - \tau(t = 12)$  of slant delay  $\tau$  (difference between  $\tau$  and its value for reference time  $t = 12$ ). The value of differences is passed in centimetres.

## Summary

The results of experiment show that for small values of the elevation angles the slant delay depends on the observation direction. It includes information concerning heterogeneity of atmospheric humidity distribution along the GPS signal path. It may be observed that in adverse weather conditions for small elevation angles ( $\xi > 3^\circ$ ), depending on the azimuth angle, the differences in time delays may be of the order of meter. More precise explanation of the obtained slant delay distribution heterogeneousness requires further research related with e.g. increasing the number of computational levels in the lower part of the model troposphere. It is planned to include in the module subroutines for satellite and radar images analysis for further research of the relation between the slant delay and the atmospheric conditions related especially with cloud systems of atmospheric fronts. The research indicates that high resolution non-hydrostatic mesoscale models like COAMPS or WRF (The Weather Research & Forecasting Model) enable to locate anisotropic distribution of water in its various phases. At the same time, they enable to determine heterogeneity of atmospheric refraction fields which can be used to investigate and simulate propagation of microwave radiation in the atmosphere. At the current phase of the project, a prototype module for slant delay determination (analysis of temporal and spatial corrections distribution of GPS signals propagation times) using data from the COAMPS non-hydrostatic mesoscale model was constructed as a result of the research. The module enables to investigate angular characteristics of the delay for various atmospheric conditions. The delay may be determined for any GPS station location in the working area of the model and any elevation angle of a satellite observed from the point. This work is sponsored by the Ministry of Science and Higher Education, Poland. Grant No NN526 2307 33.

## References

1. Niell, A.E, (2001). Preliminary evaluation of atmospheric mapping functions based on numerical weather models. *Physics and Chemistry of the Earth A*(26), pp. 475-480.

2. Boehm J, Werl B, Schuh H, (2006), Troposphere mapping functions for GPS and very long baseline interferometry from European Centre for Medium-Range Weather Forecasts operational analysis data, *J. Geophys. Res.*, 111.
3. Eresmaa R, Järvinen H, An observation operator for Ground-based GPS slant delays, *Tellus* (2006), 58A, 131-140.
4. Hodur, R.M., The Naval Research Laboratory's Coupled Ocean/Atmosphere Mesoscale Prediction System, *Monthly Weather Review*, 135, 1414-1430, 1997
5. COAMPS<sup>®</sup> version 3 model documentation, <http://www.nrlmry.navy.mil/coamps-web/web/home>
6. Bevis M, Businger S, Chiswell S, Hering TA, Anthes RA and co-authors. 1994. GPS meteorology: mapping zenith wet delays onto precipitable water. *J. Appl. Meteorology*. 33, 379-386.
7. Zou X, Vandenberghe F, Wang B, Gorbunov ME, and co-authors, September 27, 1999. A ray-tracing operator and its adjoint for the use of GPS/MET refraction angle measurements. *J. Geophys. Res.* Vol. 104, No. D18, 301–318.