Relative Positioning in Europe:

Influence of the GPS+Galileo Satellite Geometry

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ABSTRACT

While the quality of the signals successfully transmitted by GIOVE A, the first Galileo satellite launched at the end of last year, is currently being checked, a lot of work still has to be done before Galileo will be fully operational. Our previous work, especially within the context of the pure geometric aspect, showed that the use of the additional Galileo constellation with respect to GPS only, improves absolute positioning based on code observables with about 40% in terms of formal errors when simulating urban conditions. For relative positioning based on double difference carrier phase observables, the concept of RDOP (Relative Dilution of Precision) allowed to demonstrate that using GPS+Galileo, only half the observation time is sufficient to get similar precisions as with GPS only. These results were however obtained under error-free ideal conditions. The objective of this paper is to extend previous research by taking now different error sources into account, focusing only on results for relative positioning. The components of the error budget will first be treated separately, and will be put together afterwards when investigating the global error.

INTRODUCTION

Although we still have to wait a couple of years until Galileo, the European version of GPS, will be fully operational, several authors have investigated the improvement of this additional GNSS system on different applications. In our previous work [1], the effect of the geometry of the GPS+Galileo constellation on the precision of the positioning was analyzed. Also a comparison with the current positioning based on GPS was made. We showed that using Galileo in addition to GPS, a worldwide duplication of the number of daily visible satellites can be observed, yielding an improvement of about 30 to 40% for the Dilution of Precision (DOP). For relative positioning, the combined system reached similar formal errors as stand-alone GPS, but needing only half of the observation time, and consequently half the number of observations, in comparison with GPS only. These results were obtained in an ideal environment in which the effect of other error sources on the total error budget was neglected.

In this paper we will take a closer look at the influence of the errors degrading positioning on the previously obtained results. Avila-Rodriguez et al [4] already showed some first results for absolute positioning. In this paper we will first focus on each error source in particular, looking at possible changes yielded by the use of a combined GPS+Galileo system. Afterwards, a full error budget will be brought together to look at the so-called total User Equivalent Range Error $(UERE)$, which is a measure for the precision of single point positioning. Making the necessary calculations, an error budget for relative positioning will be derived from this initial error budget. Since no error would remain when using double differences for relative positioning, the model using single differences will be considered. Given this error budget for relative positioning, a couple of results will be calculated within the region of the EPN (EUREF Permanent Network). Finally, to look at how the introduced error sources change the quality of the relative positioning, Relative DOP $(RDOP)$ values will be used to calculate the horizontal and vertical relative positioning error.

The GPS satellite orbits have been created based on the broadcast navigation message, provided by IGS. For Galileo, we considered a constellation of 27 satellites distributed over three orbits with right ascension angles of respectively $-120°$, $0°$ and $120°$, equally spaced on these orbits by a mean anomaly of $-160°$, $-120°$, $-80°$, $-40°$, $0°$, $40°$, $80°$, $120°$ or $160°$. Other initial values for orbital parameters were: a semi-major axis of $29994km$, an inclination angle of $56°$, the eccentricity equal to 0, a rate of right ascension of $0°$ a day, the argument of perigee equal to $0°$ and finally a period of $14h04m42s$. For the calculation of the atmospheric errors, estimated values, also provided by the IGS were used.

IMPACT OF THE GPS+GALILEO SATELLITE GEOMETRY ON THE ERROR SOURCES

The Error Budget for Absolute Positioning

As well for code as for carrier phase observations, a certain number of systematic errors have to be taken into account when doing absolute positioning. Depending on their properties, those different error sources can be divided in following groups:

1) signal propagation errors
    a. ionospheric refraction
    b. tropospheric refraction
    c. multipath
2) satellite errors
    a. clock bias
    b. orbital errors
3) receiver ranging error
    a. clock bias
    b. ranging errors

The square root of the sum of squares of these individual errors, the so-called User Equivalent Range Error $(UERE)$, can be seen as a global error and as a measure of the precision for point positioning. Multiplying this value with the Position Dilution of Precision $(PDOP)$ consequently provides an approximation of the position error ([3] and [10]). The values of the previously mentioned error sources depend on whether we are dealing with code or with carrier phase observations. From now on, we will therefore only consider carrier phase observations.

Signal Propagation Errors

When we talk about atmospheric errors, two components have to be taken into account: the troposphere and the ionosphere. For both error types, the International GNSS Service (IGS) makes available estimations. For the troposphere the IGS provides us with Zenith Path Delay (ZPD) files for stations included in the IGS network. These weekly files contain values for the total ZPD. For the ionosphere, IONosphere map EXchange (IONEX) files give us values of the Vertical Total Electron Content $(VTEC)$ for a grid of points representing the earth. Both products are giving values at zenith and can be converted to the typical atmospheric errors using the appropriate mapping function which depends on the satellite elevation. Figure 1 shows the mean satellite elevation for the GPS constellation as well as for the Galileo constellation. The mean elevation represents a daily mean accounting for all visible satellites above an elevation cut off angle of $5°$. For both systems, the daily mean patterns are very similar from day-to-day and can be considered as representative, so later on, these values will also be considered when needing a yearly mean.

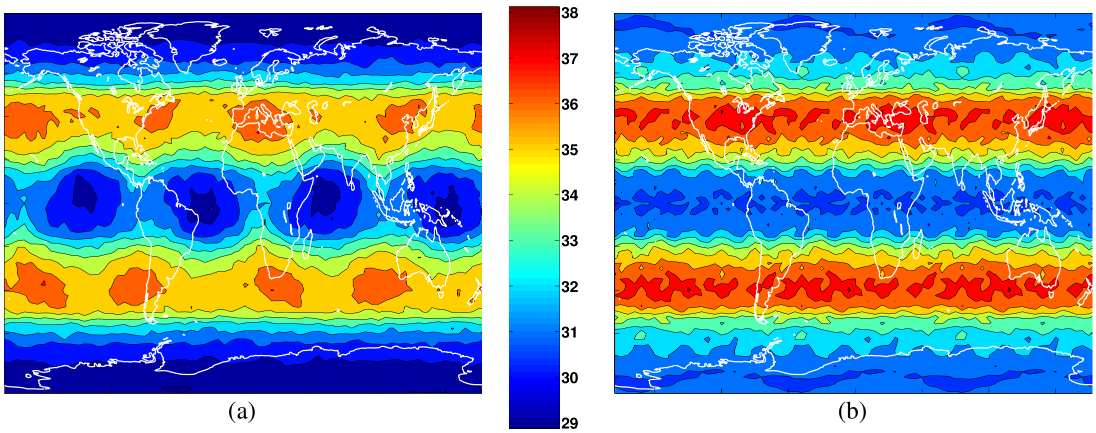


Figure 1 : Worldwide distribution of the daily mean of the elevation [°] of visible satellites using a 5° cut off,
(a) GPS only
(b) Galileo only

Figure 1 shows that for approximately 86% of the earth surface, Galileo has larger mean elevations than GPS; all the differences amount from $-2.16°$ to $2.61°$. There are no significant differences in mean elevation values between single GPS and the combined GPS+Galileo system; the elevations differences amount from $-1.30°$ to $1.06°$ worldwide and from $-0.66°$ to $0.85°$ at European level (see Figure 2).

As mentioned earlier, the IGS provides a daily ionospheric grid containing the $VTEC$ expressed in electron per square meters $[el/m^2]$ ([6] and [4]). Using the mapping function $m(elev)$ of the Klobuchar model, this $VTEC$ is mapped to the corresponding Slant Total Electron Content $(STEC)$ and the ionospheric group delay e_{iono} on L1 (in meters) can be calculated as follows:

$$e_{iono} = m(elev) * \frac{40.28}{f^2} * VTEC = \left[1 + 16(0.53 - elev)^3\right] * \frac{40.28}{f^2} * VTEC \tag{1}$$

with f the common L1 GPS & GALILEO frequency of $1575.42MHz$ and $elev$ the mean elevation of the satellites (for this model expressed in number of semicircles of $180°$).

Our results show that the new and old daily means of the ionospheric path delays, calculated respectively for the combined GPS+Galileo and for the single GPS system, are very similar. Within the European region the differences have a mean value of $-6.5mm$ and range from $-49.8mm$ to $47.4mm$. Worldwide, those values are ranging within an interval between $-14.31cm$ and $12.83cm$, with errors for the GPS-system being on average $1.35cm$ higher than those for the combined system. Highest differences are visible in vertical orientated area's around the equator (between $-20°$ and $20°$ degrees of latitude). Figure 3 shows the daily mean ionospheric path delay at respectively European and worldwide level for the combined system, with respective results of about 2.14 to $4.62m$ and of about 2.06 to $7.36m$.

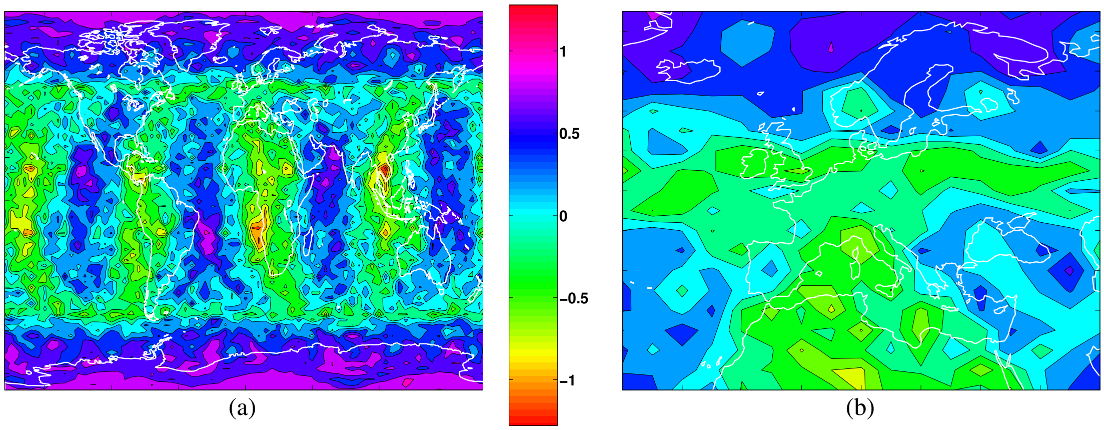


Figure 2 : Worldwide distribution of the difference in mean elevation [°] of visible satellites between the combined GPS+Galileo and the single GPS system, using a 5° cutoff
(a) worldwide
(b) for European region only

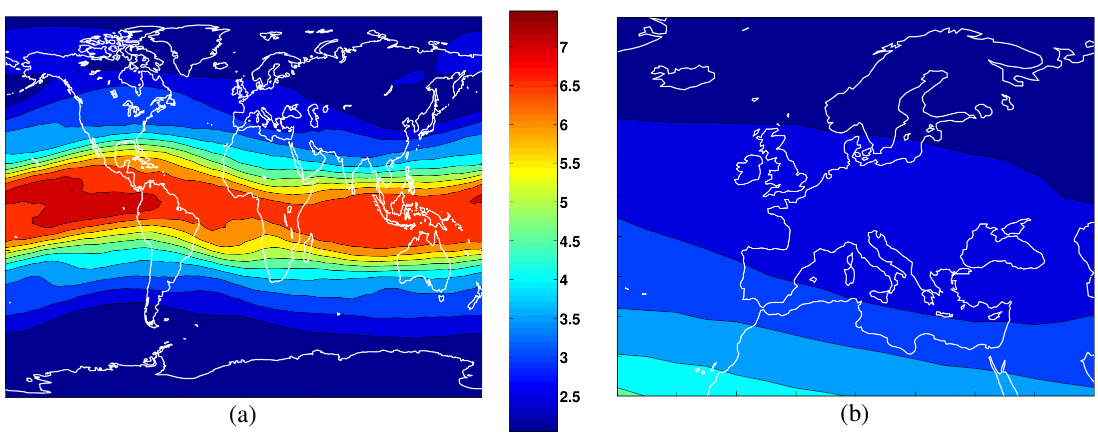


Figure 3 : Worldwide distribution of daily mean of ionospheric path delay [m] for the combined GPS+Galileo system,
(a) worldwide
(b) at European level

In the case of the troposphere, our procedure will have to be slightly adapted since the values for the total zenith path delay are only provided for some stations belonging to the IGS network. This network of stations does not contain enough data to interpolate a complete world-grid with the standard procedures for interpolation form MATLAB, but will be sufficient to make a map for the tropospheric path delay for the European region. The used mapping function is the one from Black & Eisner, suitable for both hydrostatic as well as for wet delay. This function,

$$e_{tropo} = m(elev) * e_{zenith} = \frac{1.001}{\sqrt{0.002001 + \sin^2(elev)}} * e_{zenith} \tag{2}$$

is recommended to be used for satellite elevation angles larger than 7°, and is known for its computation simplicity mainly due to the fact that no meteorological data is necessary ([4] and [5]). The mean satellite elevation values $elev$ are hereby no longer expressed in number of semicircles of $180°$, but in degrees like usual. The results show that over the European region the tropospheric error on the combined GPS+Galileo system is in average about $1.92cm$ smaller than the error on GPS only. A minimum of $-11.72cm$ and a maximum of $6.32cm$, are observed differences over Europe. Figure 4 shows the daily mean tropospheric path delay values for GPS+Galileo; they vary between 3.41 and $4.63m$ with a mean value of $4.17m$.

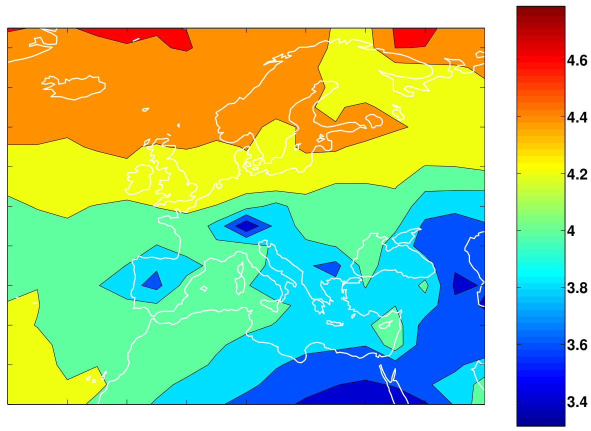


Figure 4 : Distribution of daily mean of tropospheric path delay [m] for the combined GPS+Galileo system, Europe

Finally, the last signal propagation error to discuss is the multipath error. Experimental research using the multipath characteristics of permanent GPS stations [9] showed that the phase multipath can reach up to a value of 2 to $3cm$ for the slant delay. Within the European region, we will therefore consider a common multipath error of $3cm$ for the remaining part of this paper.

Satellite and Receiver Errors

As for the case of the multipath error, for the remaining part of this paper, common values will also be assigned to all receiver and satellite errors. Based on the IGS web-pages [8] a value of $5cm$ is used for the satellite orbit error. IGS final clock products have an accuracy smaller than of $0.1ns$, equivalent with a satellite as well as a receiver clock error of $3cm$. Finally, receiver ranging errors, appearing when measuring carrier phases, will not be considered since these errors seems to be negligible (less than one millimeter) for high quality receivers [7].

Numerical Overview of the Error Sources

Table 1 gives an overview of the all the errors and their values that will be considered further on within the European region for the combined GPS+Galileo system.. The total $UERE$, i.e. the square root of the sum of squares of the individual errors, will range between $4.02m$ and $6.54m$.

	$[m]$
satellite clock error	0.05
satellite orbit error	0.03
ionospheric path delay	$2.14 - 4.62$
tropospheric path delay	$3.41 - 4.63$
multipath	0.03
receiver clock error	0.03
receiver ranging error	/
UERE	**4.02 – 6.54**

Table 1 : Overview of all the error values and total *UERE* at European level

The position error is obtained by multiplying the $UERE$ with the $PDOP$. Figure 5 shows European maps of the horizontal and vertical position error for single GPS system as well as for the combined GPS+Galileo system.

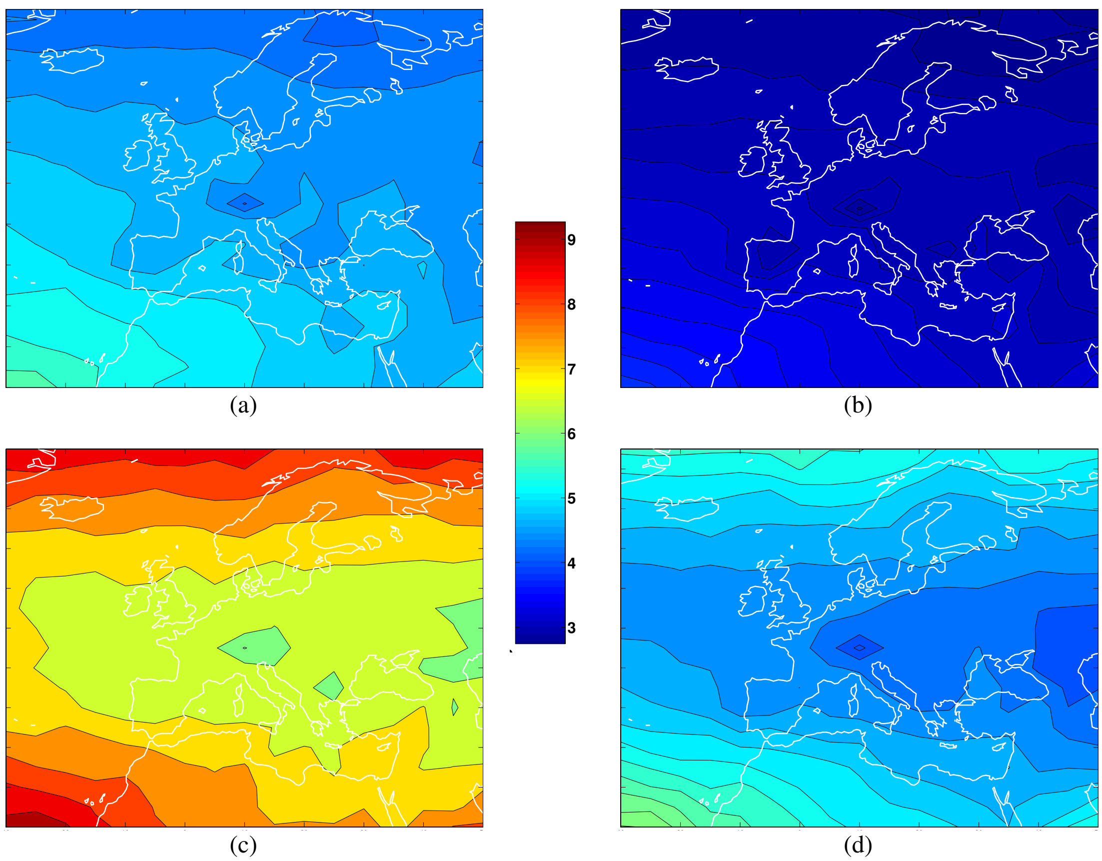
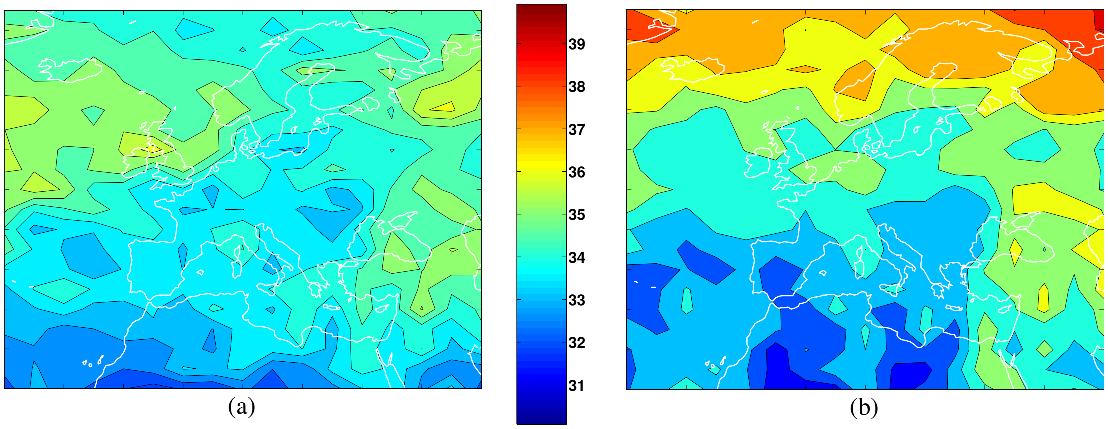


Figure 5 : Distribution of daily mean positioning error [*m*] at European level,
(a) horizontal position error, single GPS system
(b) horizontal position error, combined GPS+Galileo system
(c) vertical position error, single GPS system
(d) vertical position error, combined GPS+Galileo system

The results in Figure 5a for the GPS system show horizontal positioning errors ranging between 4.09 and $5.71m$, while for the combined system, Figure 5b, equivalent values are all below $3.87m$. Figure

5c and Figure 5d show similar results for the case of the vertical positioning errors, with values ranging from 5.97 to $9.32m$ for the GPS system, while equivalent values for the combined system are all below $6.22m$. For both systems as well in the case of vertical as horizontal positioning errors, we observe in both cases a mean improvement of about 34% for the European region, ranging between 30 and 40%. This improvement for the European region is showed in Figure 6.



(a) (b)

Figure 6 : distribution of improvement ratio [%] for horizontal (a) and vertical (b) positioning error

RELATIVE POSITIONING

Single Difference Carrier Phase Model for Relative Positioning

Calculating the vector $(X_{pq}, Y_{pq}, Z_{pq}) = (X_p - X_q, Y_p - Y_q, Z_p - Z_q)$ between two receivers p and q instead of one single receiver position is the main difference between absolute and relative positioning. Solving this model requires the use of observables as single (SD) or as double differences (DD), yielding to a decrease or elimination of the influence of some error sources.
In this paper, we replace the DD model by SD because, using our assumptions, the DD model would lead to a full elimination of all errors. The model for relative positioning using SD of the observables (3) will therefore be used for the remaining part of this paper.

$$\Phi_{pq}^j = \Phi_q^j - \Phi_p^j = \rho_{pq}^j + c\delta_{pq} + \lambda A_{pq}^j - I_{pq}^j + T_{pq}^j + MP_{pq}^j + \varepsilon_{\Phi_{pq}^j} \tag{3}$$

Φ_{pq}^j is the SD of the carrier phase observable, ρ_{pq}^j the SD of the approximate geometric distances between receivers p and q and satellite j, λ and c the respective wavelength and speed (= *speed of light*) of the signal, while A_{pq}^j finally is the SD non-integer ambiguity term. Remaining terms of equation (3) δ_{pq}, I_{pq}^j, T_{pq}^j, and MP_{pq}^j are the SD of the error terms, representing respectively receiver clock error, ionosphere path delay, troposphere path delay and multipath. As for absolute positioning, an a priori estimated position (X_{0q}, Y_{0q}, Z_{0q}) for the unknown position of receiver q is given, allowing to rewrite this unknown position as $(X_q, Y_q, Z_q) = (X_{0q} + \Delta X_q, Y_{0q} + \Delta Y_q, Z_{0q} + \Delta Z_q)$. Rewriting the known position of reference receiver p the same way, Taylor series expansion will afterwards be executed around $(\Delta X_p, \Delta Y_p, \Delta Z_p)$ and $(\Delta X_q, \Delta Y_q, \Delta Z_q)$ for respective terms belonging to given receivers p and q. Receiver p being the reference station with known coordinates, will consequently yield to a simplification of the model equation since $\Delta X_p = \Delta Y_p = \Delta Z_p = 0$.

This way, equation (3) will be linearized in order to solve for the unknown parameters:

$$\Phi_{pq}^{j} - c\delta_{pq} - \lambda A_{pq}^{j} + I_{pq}^{j} - T_{pq}^{j} - MP_{pq}^{j} = \rho_{0pq}^{j} - \frac{X^{j} - X_{0q}}{\rho_{0q}^{j}} \Delta X_{q} - \frac{Y^{j} - Y_{0q}}{\rho_{0q}^{j}} \Delta Y_{q} - \frac{Z^{j} - Z_{0q}}{\rho_{0q}^{i}} \Delta Z_{q} \tag{4}$$

Every observation can be written as equation (4), yielding to a model represented by the matrix equation $L = AX + v$, with X as the vector $(\Delta X_q, \Delta Y_q, \Delta Z_q)$ of the unknowns, A the design matrix containing all coefficients of the unknowns, L the vector containing all observations and finally v as the vector of residuals. Using this observation model we can compute the associated covariance matrix of the unknowns $\Sigma_X = (A^T \Sigma_L^{-1} A)^{-1}$ and convert it to its topocentric equivalent Σ_T. The covariance matrix of the observables Σ_L is not a unit matrix, but the mathematical correlations between the SD measurements are taken into account. This covariance matrix Σ_X of the unknowns provides information on the precision of the solution. The $RDOP$ (Relative DOP) is similar to the $PDOP$ value for the case of absolute positioning, but will be calculated in a different way with the formula, (4):

$$RDOP = \sqrt{\frac{trace(\Sigma_X)}{\sigma_{SD}^2}} \tag{5}$$

Contrary to the computation of the $PDOP$, the observations are accumulated over sessions varying between $1/2$ and 24 hours, using a 60-second measurement interval. Going back to equation (5), σ_{SD} is the uncertainty of a SD measurement. This definition implies that the $RDOP$ will not depend on the a priori variance σ^2 of the carrier phase measurements, since for the matrix Σ_X as well as for the value σ_{SD}^2, a factor σ^2 can be set apart, whereas those factors appearing in denominator and nominator of equation (5) can be removed. Consequently, we will not have to make assumptions about this value. The units of $RDOP$ are meters/cycle. Theoretically, the uncertainty of a SD measurement multiplied by $RDOP$ will therefore yield a relative position error, and a closer look at this error will be taken.

Error Budget of the Single Difference Carrier Phase Model

All SD between receivers p and q are calculated as $\bullet_{pq}^{j} = \bullet_{q}^{j} - \bullet_{p}^{j}$. Consequently, a property of using SD, is the elimination of the satellite clock error within the model. Because of the use of fixed representative values for receiver clock error and multipath, these errors are also eliminated using SD and will not have to be taken into account for the computation of the uncertainty of a SD measurement. In comparison with the errors appearing in Table 1, only the satellite orbit error and ionospheric and tropospheric errors will therefore be used. For the atmospheric errors, their SD will now be considered, while the orbital error of a satellite for the case of relative positioning will be equal to its equivalent error for the case of absolute positioning, multiplied by $d/20000$ with d the baseline length between receiver p and q expressed in kilometers, [6]. After elimination of some errors conform with SD properties, and other errors yielded by the use representative values for them used in this paper, the error budget for relative positioning, as we assume it here, will finally consist of:

1) SD of signal propagation errors
    a. ionospheric refraction
    b. tropospheric refraction
2) satellite orbital errors

Set-up of Calculations and Results

Following the same principle as for $HDOP$ and $VDOP$, we considered horizontal and vertical components for the $RDOP$ values. Those values are calculated for several baselines between EPN stations, as shown in Figure 7**,** subdivided in three groups depending on their orientation.

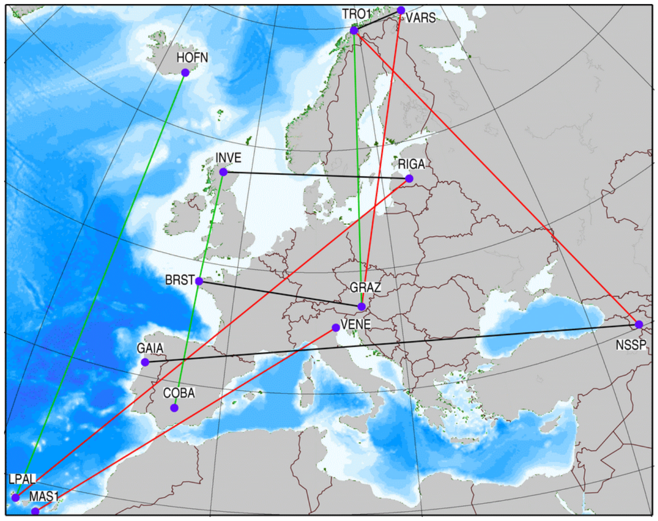


Figure 7 : Map of Europe showing considered baselines

The results show that all observations accumulated over sessions of 12 hours, using a SD model, show the same improvement of 30% for horizontal as well as for vertical components in comparison with GPS only. Note that the errors for horizontal baselines, i.e. baselines between stations with equal latitudes, are systematically less than those for the vertical as well as for the diagonal baselines. This is mainly due by the fact that for these horizontal baselines, atmospheric errors of both baseline stations don't often differ much. Nevertheless, the improvement of 30% was similar for all kinds of baselines. This improvement is shown in Figure 8b where the $RDOP$ improvement ratio is shown for horizontal and vertical components all together. The magenta and red colored lines in Figure 8a represent the respective $RDOP$ values for the combined GPS+Galileo and for the single GPS system.

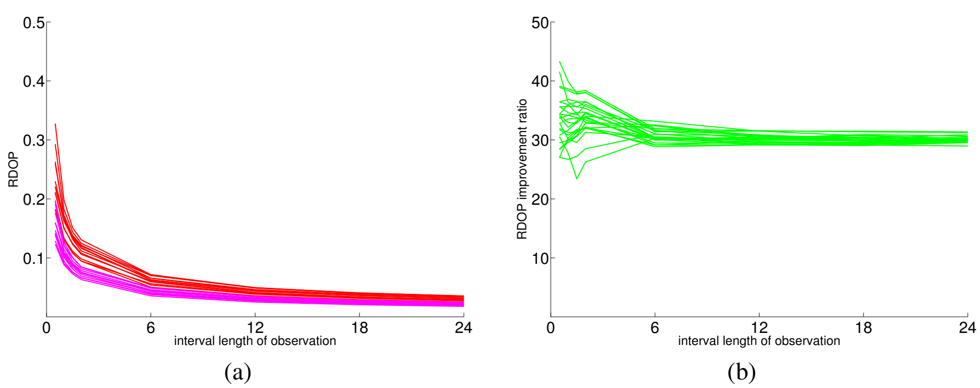


Figure 8 :
(a) *RDOP* values for single GPS (red) and combined GPS+Galileo (magenta) system
(b) *RDOP* improvement ratio, horizontal and vertical components put together

The position errors were considered separately for the horizontal and vertical components; Figure 9, also show an improvement of about 30% for all components of the position error.

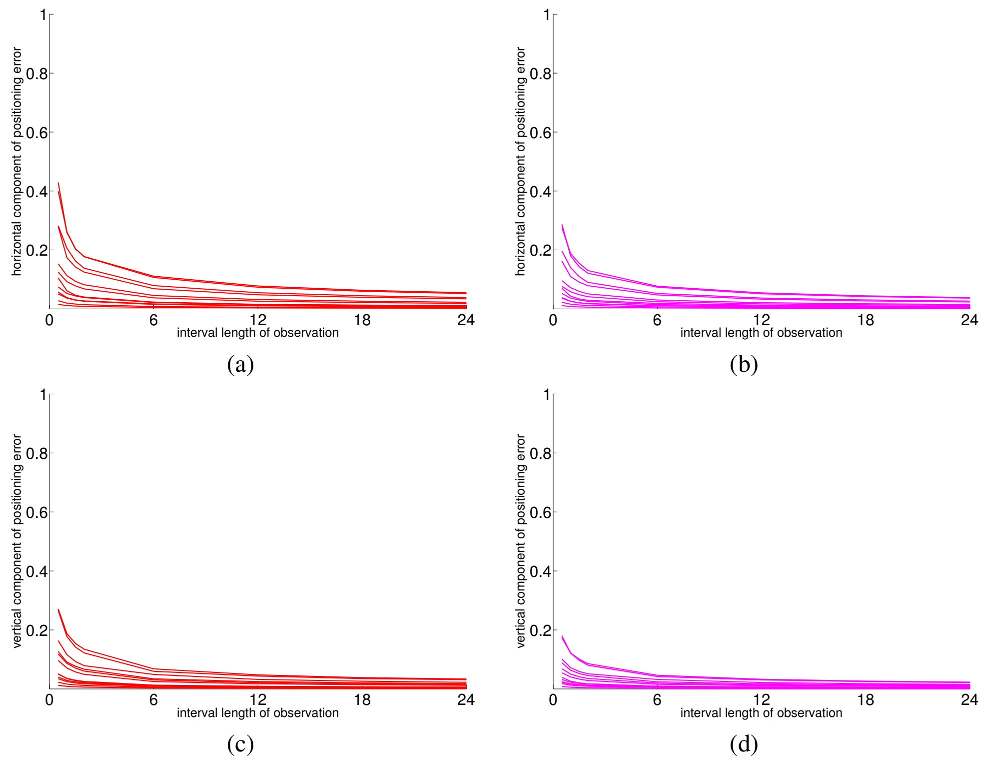


Figure 9 : Componential Relative Positioning error [m] for a given system
(a) horizontal component of position error, single GPS system
(b) horizontal component of position error, combined GPS+Galileo system
(c) vertical component of position error, single GPS system
(d) vertical component of position error, combined GPS+Galileo system

CONCLUSION

This paper compared the theoretical positoning error of GPS only with the error of the future GPS+Galileo combined system. Using IGS products to compute atmospheric errors with adequate mapping functions, no big changes were observed for the values of these errors when considering them individually. Nevertheless, the improvement in DOP values, showed in previous work [1], imply a similar improvement for the case of the approximate positioning error. This is true as well for the case of absolute positioning considering $PDOP$ values, with an improvement ranging from 30 to 40%, as for the case of relative positioning with SD considering $RDOP$ values, with an improvement of about 30%.

REFERENCES

[1] S. Daghay, M. Moins, C. Bruyninx, Y. Rolain, F. Roosbeek
Impact of the Combined GPS+GALILEO Satellite Geometry on Positioning Precision,
Proceedings of EUREF 2005, 1-3 June 2005, Vienna, Austria (in press).

[2] C.C. Goad
Investigation Of An Alternate Method Of Processing Global Positioning Survey Data Collected In Kinematic Mode,
GPS-Techniques Applied to Geodesy and Surveying, April 1988.

[3] G.J. Husti
Global Positioning System een inleiding,
Series on Mathematical Geodesy and Positioning, Delft University Press, 2000.

[4] J.A. Avila-Rodriguez, G.W. Hein, S. Wallner, T. Schueler, E. Schueler, M. Irsigler
Revised Combined Galileo/GPS Frequency and Signal Performance Analysis,
Proceedings of ION GNSS 2005, 13-16 September 2005, Long Beach, California, USA.

[5] J. Guo, R.B. Langley
A New Tropospheric Propagation Delay Mapping Function for Elevation Angles Down to 2°,
Proceedings of ION GNSS 2003, 9-12 September 2003, Portland, Oregon, USA.

[6] C. Rizos
Principles and Practice of GPS Surveying,
http://www.gmat.unsw.edu.au/snap/gps/gps_survey/principles_gps.htm ,
The University of New South Wales, Sidney, Australia, 1999.

[7] J. Ashjaee, N. Ashjaee
Basics of High-Precision Global Positioning System,
http://www.javad.com/index.html?/jns/gpstutorial/ ,
Javad Navigation Systems, San Jose, California, USA, 1998.

[8] *IGS Product Table,*
http://igscb.jpl.nasa.gov/components/prods.html ,
International GNSS Service, 2005.

[9] H. van der Marel
Computation and Analysis of Antenna and Multipath Characteristics of Permanent GPS Stations
Proceedings of IGS Workshop, 8 May 2006, Darmstadt, Germany (in press)

[10] G. Strang, K. Borre
Linear Algebra, Geodesy and GPS,
Wellesley – Cambridge Press, 1997.